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Three Vertex and Parallelograms in the Affine Plane: Similarity and Addition Abelian Groups of Similarly *n*-Vertexes in the Desargues Affine Plane

Orgest Zaka

Department of Mathematics, Faculty of Technical Science, University "Ismail QEMALI"of Vlora, Vlora, Albania

Email address:
gertizaka@yahoo.com

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Abstract: In this article will do a' concept generalization *n*-gon. By renouncing the metrics in much axiomatic geometry, the need arises for a new label to this concept. In this paper will use the meaning of *n*-vertexes. As you know in affine and projective plane simply set of points, blocks and incidence relation, which is argued in [1], [2], [3]. In this paper will focus on affine plane. Will describe the meaning of the similarity *n*-vertexes. Will determine the addition of similar three-vertexes in Desargues affine plane, which is argued in [1], [2], [3], and show that this set of three-vertexes forms an commutative group associated with additions of three-vertexes. At the end of this paperare making a generalization of the meeting of similarity *n*-vertexes in Desargues affine plane, also here it turns out to have a commutative group, associated with additions of similarity *n*-vertexes.

Keyword: *n*-vertexes, Desargues Affine Plane, Similarity of *n*-Vertexes, Abelian Group

1. Introduction

In Euclidian geometry use the term three-angle and non three-vertex, this because the fact that the Euclidean geometry think of associated with metrics, which are argued in [4], [6], [7]. In this paper will use the "three-Vertex" term, by renouncing the metric. Will generalize so its own meaning in the Euclidean case. With the help of parallelism [1], [2], [3] will give meaning of similarity and will see that have a generalization of the similarity of the figures in the Euclidean plane. By following the logic of additions of points in a line of Desargues affine plane submitted to [3], herewill show that analogously this meaning may also extend to the addition of similarity three-vertex in Desargues affine plane, moreover extend this concept for the similarity *n*-vertexes to the Desargues affine plane.

The aim is to see if the move to three-vertexes as well as to *n*-vertexes has the group's properties, which are arguing that the best in [5], [8], [9].

2. *n*-Vertexes in Affine Plane and Their Similarity

2.1. 3-Vertexes and Their Similarity

Let's have the affine plane $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$.

Definition 2.1.1 *Three-Vertex will called an ordered trio of non-collinear points (A,B,C) in an affine plane.*

Definition 2.1.2 *Two three-vertexes* (A_1,B_1,C_1) *and* (A_2,B_2,C_2) *will call similar if they meet conditions:*

$$A_1B_1//A_2B_2;\ A_1C_1//A_2C_2 \text{ and } B_1C_1//B_2C_2$$

Example 2.1.1 In affine plane of the second order have the similar three-vertices (Figure 1):

$$(A,D,C)\approx(B,C,D): \text{because,}$$
$$AD\parallel BC; DC\parallel CD; AC\parallel BD$$

$$(A,B,D)\approx(C,D,B): \text{because,}$$
$$AB\parallel CD; BD\parallel DB; AD\parallel CB$$

$$(A,B,D) \approx (D,C,A): \text{because,}$$
$$AB \parallel DC; BD \parallel CA; AD \parallel DA$$

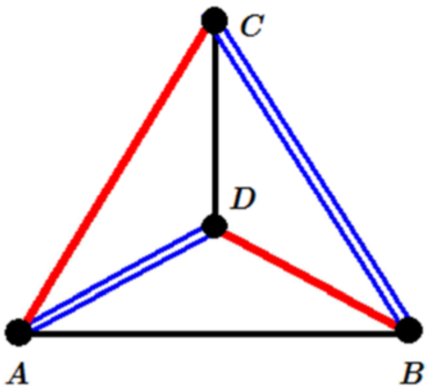

Figure 1. The similar three-vertexes in affine plane of order 2.

Example 2.1.2: In the third order affine plane.

$$(2,3,9) \approx (7,9,3), \text{because:}$$
$$\ell_{(2,3)} \parallel \ell_{(7,9)};\ \ell_{(2,9)} \parallel \ell_{(7,3)};\ \ell_{(3,9)} \parallel \ell_{(9,3)};$$

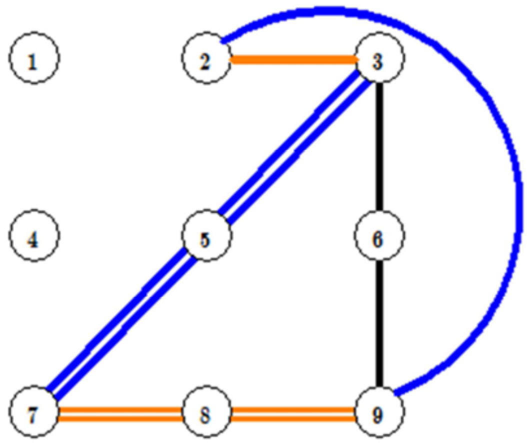

Figure 2. Two similar three-vertexes in affine plane of order 3.

Proposition 2.1.1: The similarity of the three-vertexes is equivalence relation.

Proof: **1)** It is clear that every three-vertexes (A,B,C) is similar to yourself.

$$(A,B,C) \approx (A,B,C)$$

2) If three-vertexes $(A_1,B_1,C_1) \approx (A_2,B_2,C_2)$, are similar then also three-vertexes $(A_2,B_2,C_2) \approx (A_1,B_1,C_1)$, are similarity since from:

$$A_1B_1//A_2B_2;\ A_1C_1//A_2C_2;\ B_1C_1//B_2C_2 \Longrightarrow$$
$$\Longrightarrow A_2B_2//A_1B_1;\ A_2C_2//A_1C_1;\ B_2C_2//B_1C_1.$$

3) If $(A_1,B_1,C_1) \approx (A_2,B_2,C_2)$, and three-vertexes $(A_2,B_2,C_2) \approx (A_3,B_3,C_3)$ then have to $(A_1,B_1,C_1) \approx (A_3,B_3,C_3)$, because parallelism in the affine plane is equivalence relation, which is described in [2], [3], [4].

So would have to:

$$A_1B_1//A_2B_2;\ A_1C_1//A_2C_2;\ B_1C_1//B_2C_2$$

and

$$A_2B_2//A_3B_3;\ A_2C_2//A_3C_3;\ B_2C_2//B_3C_3$$

since the parallelism in the affine plane is equivalence relation then will have to:

$$A_1B_1//A_2B_2 \text{ and } A_2B_2//A_3B_3 \Longrightarrow A_1B_1//A_3B_3;$$
$$A_1C_1//A_2C_2 \text{ and } A_2C_2//A_3C_3 \Longrightarrow A_1C_1//A_3C_3;$$
$$B_1C_1//B_2C_2 \text{ and } B_2C_2//B_3C_3 \Longrightarrow B_1C_1//B_3C_3.$$

Well,

$$\boldsymbol{(A_1,B_1,C_1) \approx (A_3,B_3,C_3)}.$$

2.2. 4-Vertexes

Definition 2.2.1: *In affine plane $\mathcal{A}$, a set of four-point three out of three not-collineary will call **4-vertexes**.*

Definition 2.2.2: *Two 4-vertexes ABCD and A'B'C'D' will call similar only if have the following parallels:*

AB||A'B', BC||B'C', CD||C'D' and DA||D'A'.

2.3. Parallelograms

Definition 2.1.3: *Parallelogram will call the ordered quartet of points (A,B,C,D) from $\mathcal{P}$, that meets the conditions: AB//CD and BC//AD the lines AC and BD are called the diagonal of parallelogram.*

Example 2.2.1: In affine plane of the second order (Figure 3. a.) have the following parallelogram:

(A,D,B,C) with the diagonal AB and DC (Figure 3. b);
(A,B,D,C) with the diagonal AD and BC (Figure 3. c);
(A,B,C,D) with the diagonal AC and BD (Figure 3. d).

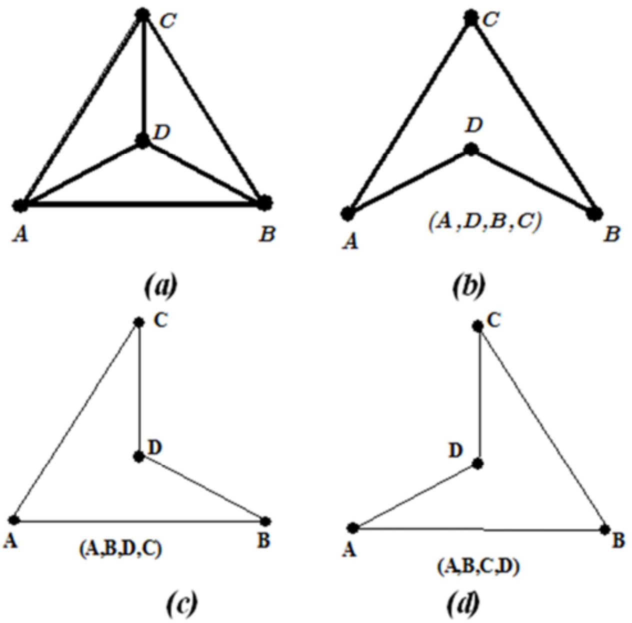

Figure 3. 4-parallelograms in the affine plane of order 2.

From the definition of parallelogram and the fact that parallelism is the equivalence relation is evident this Proposition:

Proposition 2.2.1: *If you have two similar 4-vertexes, where each is parallelogram then another 4-vertexes will be parallelogram.*

2.4. n-Vertexes

Definition 2.4.1: *In affine plane $\mathcal{A}$, a set of n-points non-*

collinearly three out of three will call n-vertex.

Definition 2.4.2: *Two n-vertexes ($A_1A_2...A_n$) and ($B_1B_2...B_n$) will call similar just if have the following parallelisms:*

$$\mathbf{A_iA_j \parallel B_iB_j},$$

$$\forall(i,j)\in \{(1,1),\ldots,(1,n);(2,1),\ldots,(2,n);\ldots;(n,1),\ldots,(n,n)\}.$$

3. The Addition of Similarity Three-Vertexes in the Desargues Affine Plane

Let's have two similarity three-vertexes (A_1,A_2,A_3) and (B_1,B_2,B_3) in the Desargues affine plane $\mathcal{A}_D=(\mathcal{P},\mathcal{L},\mathcal{I})$. Constructed the lines A_1B_1, A_2B_2, A_3B_3, since are in Desargues affine plane and the similarity of three-vertexes have to: $A_1A_2 \| B_1B_2$; $A_2A_3 \| B_2B_3$; $A_1A_3 \| B_1B_3 \Rightarrow$ the lines A_1B_1, A_2B_2 and A_3B_3, or will be parallel or will cross the on a single point. Receive now a point $O_1\in A_1B_1$, and find points O_2 and O_3 how:

$$O_2 = A_2B_2 \cap \ell^{O_1}_{A_1A_2} \text{ and } O_3 = A_3B_3 \cap \ell^{O_1}_{A_1A_3}$$

So have obtained thus three-vertexes (O_1,O_2,O_3), (points O_1, O_2 and O_3 are non-collinearly, because from construction this three-vertexes will be similar with three-vertexes (A_1,A_2,A_3)) where $O_1\in A_1B_1$, $O_2\in A_2B_2$ and $O_3\in A_3B_3$. This three-vertex called 'zero' three-vertex. So have three lines, to which each have its zero point. Now just as to [3], additions of the points of each line based on the algorithm of additions of points in a line in Desargues affine plans, and take:

$$\mathbf{C_1=A_1+B_1,\ C_2=A_2+B_2,\ C_3=A_3+B_3.} \quad (1)$$

Definition 3.1: *The addition of two similarity three-vertexes* **(A_1,A_2,A_3)** *and* **(B_1,B_2,B_3)**,*called three-vertexes* **(C_1,C_2,C_3)**, *where the points (vertexes)* **C_1,C_2,C_3.** *They found according to equation* (1) (Figure 4).

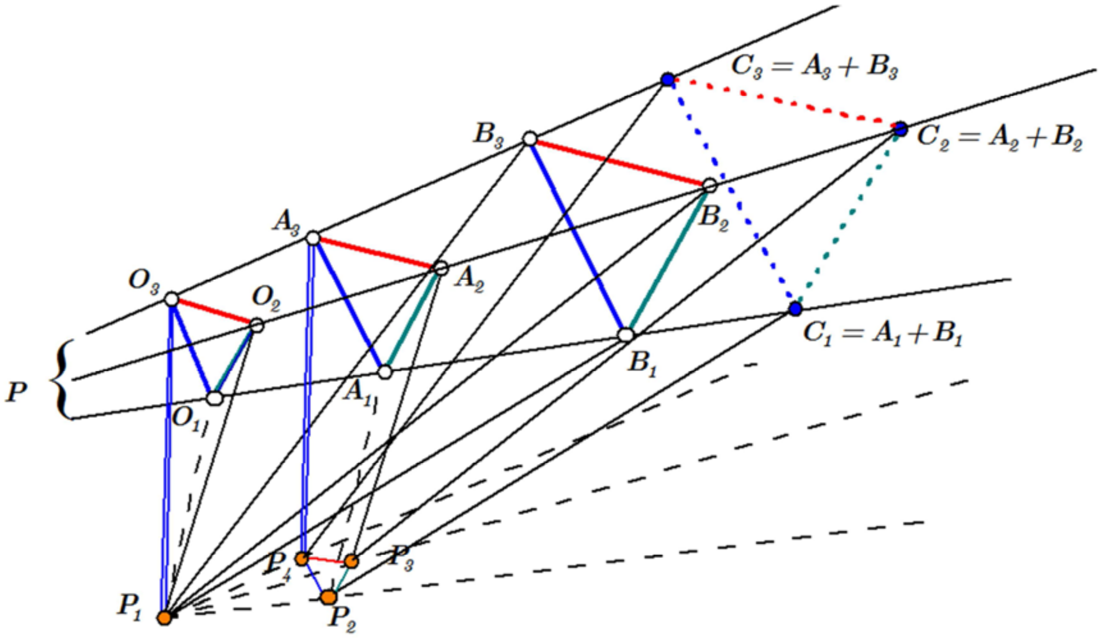

Figure 4. *The Addition of two similarity three-Vertexes in the Desargues Affine Plane.*

From construction of three-vertexes as the addition of two similar three-vertexes have evident this Proposition:

Proposition 3.1 *Three-vertexes that obtained as the sum of two similar three-vertexes (A_1,A_2,A_3) and (B_1,B_2,B_3), it is similar to the first two.*

Well

$$(A_1+B_1, A_2+B_2, A_3+B_3)\approx(A_1,A_2,A_3)$$

and

$$(A_1+B_1, A_2+B_2, A_3+B_3)\approx(B_1,B_2,B_3)$$

Proposition 3.2 *The additions of non-similarity three-vertexes it may not be a three-vertexes.*

Proof: If renounce above from addition algorithm of the similar three-vertexes. In the same logic, are additions together two of whatever three-vertexes. Let's have two whatever three-vertexes (A_1,A_2,A_3) and (B_1,B_2,B_3) in the affine plane $\mathcal{A}=(\mathcal{P},\mathcal{L},\mathcal{I})$. Construct the line A_1B_1, A_2B_2 and A_3B_3. Get a whatever three-vertexes (O_1,O_2,O_3) (the points O_1, O_2 and O_3 are non-collinearly) where $O_1\in A_1B_1$, $O_2\in A_2B_2$ and $O_3\in A_3B_3$. This three-vertexes called the 'zero' three-vertex. So have three lines, where, in each line have hers zero point. Now just as to [3], the addition points of every line based on the addition algorithm given to [3], and take: $C_1=A_1+B_1$, $C_2=A_2+B_2$ and $C_3=A_3+B_3$.

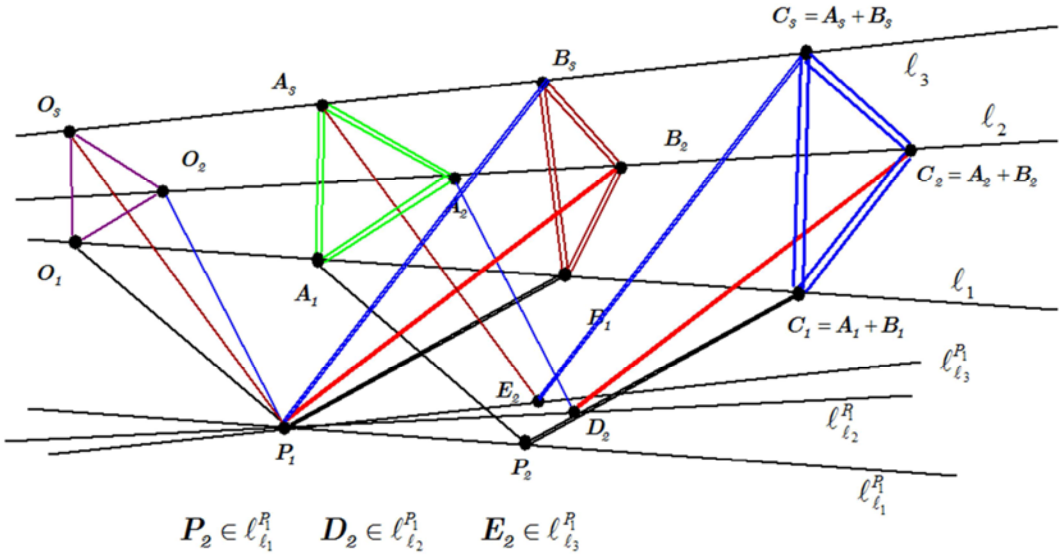

Figure 5. *The Addition of two non-similarity three-Vertexes in the Desargues Affine Plane is a three-Vertex.*

Defined in this way, it seems as if does not have a contradiction. But the veracity of this Proposition are presenting with the help of a simple anti-example, shown in the following figure.

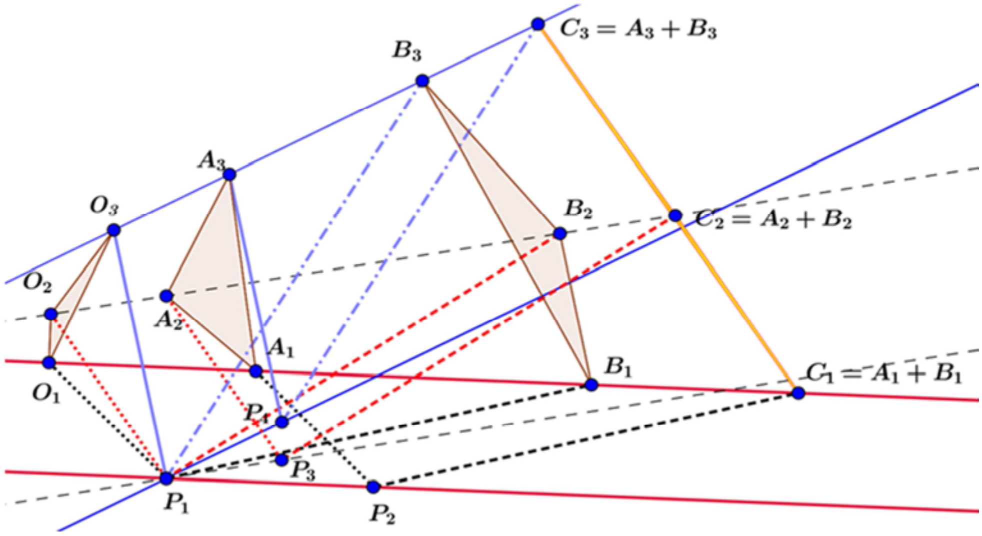

Figure 6. *The Addition of two non-similarity three-Vertexes in the Desargues Affine Plane is not a three-Vertex.*

Remark 3.1: By following the addition algorithms for points in a line of Desargues affine plane, is sufficient to get only an auxiliary point P_1, for this obedient from [3], for three sums can either take one three-vertexes (P_1,P_2,P_3), wherein each point of three-vertexes be auxiliary point for the relevant sum.

Remark 3.2: Marked the set of three-vertexes in the Desargues affine plans with symbol $\mathcal{T}^{\mathcal{D}.Aff}$.

Remark 3.3: Marked the set of *similarity* three-vertexes in the Desargues affine plans with symbol $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$.

It is clear that: $\mathcal{T}_{\approx}^{\mathcal{D}.Aff} \subset \mathcal{T}^{\mathcal{D}.Aff}$.

Let us be (A_1,A_2,A_3) and (B_1,B_2,B_3) two whatsoever three-vertexes in the set $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$. I associate pairs

$$\left[(A_1,A_2,A_3),(B_1,B_2,B_3)\right] \in \mathcal{T}_{\approx}^{\mathcal{D}.Aff} \times \mathcal{T}_{\approx}^{\mathcal{D}.Aff},$$

three-vertex $(C_1,C_2,C_3) \in \mathcal{T}_{\approx}^{\mathcal{D}.Aff}$, that the vertexes are determines with algorithm in [3]. According to the preceding Theorems, three-vertexes (C_1,C_2,C_3) is determined in single mode by [3].
Thus obtain an application

$$\mathcal{T}_{\approx}^{\mathcal{D}.Aff} \times \mathcal{T}_{\approx}^{\mathcal{D}.Aff} \to \mathcal{T}_{\approx}^{\mathcal{D}.Aff}.$$

Definition 3.2: *In the above conditions, application*

$$+ : \mathcal{T}_{\approx}^{\mathcal{D}.Aff} \times \mathcal{T}_{\approx}^{\mathcal{D}.Aff} \to \mathcal{T}_{\approx}^{\mathcal{D}.Aff},$$

defined by

$$\left[(A_1,A_2,A_3),(B_1,B_2,B_3)\right]\mapsto(C_1,C_2,C_3)$$
$$\forall\left[(A_1,A_2,A_3),(B_1,B_2,B_3)\right]\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}\times\mathcal{T}_{\approx}^{\mathcal{D}.Aff}.$$

The addition in $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ according to this Definitions, can write

$$\forall(A_1,A_2,A_3),(B_1,B_2,B_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff},$$

$$\left.\begin{array}{l}1.\ P_1\notin A_1B_1,A_2B_2,A_3B_3,\\ 2.\ \ell_{A_1B_1}^{P_1}\cap\ell_{O_1P_1}^{A_1}=P_2,\\ 3.\ \ell_{P_1B_1}^{P_2}\cap A_1B_1=C_1.\\ 4.\ \ell_{A_2B_2}^{P_1}\cap\ell_{O_2P_1}^{A_2}=P_3,\\ 5.\ \ell_{P_1B_2}^{P_3}\cap A_2B_2=C_2.\\ 6.\ \ell_{A_3B_3}^{P_1}\cap\ell_{O_3P_1}^{A_3}=P_4,\\ 7.\ \ell_{P_1B_3}^{P_4}\cap A_3B_3=C_3.\end{array}\right]\Leftrightarrow \tag{2}$$
$$\Leftrightarrow(A_1,A_2,A_3)+(B_1,B_2,B_3)=(C_1,C_2,C_3).$$

Theorem 3.1: *For every two three-vertexes* (A_1,A_2,A_3), $(B_1,B_2,B_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ *, algorithm (2) determines the single three-vertexes* $(C_1,C_2,C_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ *, which does* ***not depend*** *on the choice of hers auxiliary point* P_1.

Proof: From Theorem 2.1, in [3], have to addition of two points in a line of Desargues affine plane does not depend on the choice of hers auxiliary point. For this reason keep as auxiliary points for addition of pairs points, the auxiliary point P_1.

From Theorem 3.1, appears immediately true this

Proposition 3.3: *Additions of three-vertexes in* $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ *there are element* ***zero*** *the three-vertexes* (O_1,O_2,O_3):

$$\begin{aligned}&\forall(A_1,A_2,A_3)\in\mathcal{T}^{\mathcal{D}.Aff},\\&(A_1,A_2,A_3)+(O_1,O_2,O_3)=\\&=(O_1,O_2,O_3)+(A_1,A_2,A_3)=\\&=(A_1,A_2,A_3)\end{aligned}\tag{3}$$

As well as worth and below Propositions.

Proposition 3.4: *Additions of three-vertexes is* ***commutative*** *in* $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ *:*

$$\begin{aligned}&\forall(A_1,A_2,A_3),(B_1,B_2,B_3)\in\mathcal{T}^{\mathcal{D}.Aff}\\&(A_1,A_2,A_3)+(B_1,B_2,B_3)=\\&=(B_1,B_2,B_3)+(A_1,A_2,A_3).\end{aligned}\tag{4}$$

Proof: By definition of additions of three-vertexes that have:

$$(A_1,A_2,A_3)+(B_1,B_2,B_3)=(A_1+B_1,A_2+B_2,A_3+B_3)$$

From Theorem 2.1, in [3], have that for every two points is a line in the Desargues affine plane the addition is commutative, and consequently have to:

$$(A_1,A_2,A_3)+(B_1,B_2,B_3)=(A_1+B_1,A_2+B_2,A_3+B_3)$$
$$\overset{[1]}{=}(B_1+A_1,B_2+A_2,B_3+A_3)=(B_1,B_2,B_3)+(A_1,A_2,A_3).$$

Proposition 3.5: *Addition of three-vertexes is* ***associative*** *in* $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ *:*

$$\begin{aligned}&\forall(A_1,A_2,A_3),(B_1,B_2,B_3),(C_1,C_2,C_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}\\&(A_1,A_2,A_3)+\left[(B_1,B_2,B_3)+(C_1,C_2,C_3)\right]=\\&=\left[(A_1,A_2,A_3)+(B_1,B_2,B_3)\right]+(C_1,C_2,C_3).\end{aligned}\tag{5}$$

Proof: Let's have three whatever three-vertexes

$$(A_1,A_2,A_3),(B_1,B_2,B_3),(C_1,C_2,C_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$$

Appreciate now,

$$\begin{aligned}&(A_1,A_2,A_3)+\left[(B_1,B_2,B_3)+(C_1,C_2,C_3)\right]=\\&=(A_1,A_2,A_3)+(B_1+C_1,B_2+C_2,B_3+C_3)\\&=\left[A_1+(B_1+C_1),A_2+(B_2+C_2),A_3+(B_3+C_3)\right]\\&\overset{[1]}{=}\left[(A_1+B_1)+C_1,(A_2+B_2)+C_2,(A_3+B_3)+C_3\right]\\&=(A_1+B_1,A_2+B_2,A_3+B_3)+(C_1,C_2,C_3)\\&=\left[(A_1,A_2,A_3)+(B_1,B_2,B_3)\right]+(C_1,C_2,C_3).\end{aligned}$$

Proposition 3.6: *For every three-vertex in* $\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ *exists her* ***right symmetrical*** *according to addition:*

$$\forall(A_1,A_2,A_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff},\exists\overline{(A_1,A_2,A_3)}\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff},$$
$$(A_1,A_2,A_3)+\overline{(A_1,A_2,A_3)}=(O_1,O_2,O_3)\tag{6}$$

Proof: Let us have whatever $(A_1,A_2,A_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$, fix the 'zero' three-vertexes $(O_1,O_2,O_3)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ (which would be similar to three-vertexes (A_1,A_2,A_3)) if apply the Proposition 3. 4, in [3] pp34990, have that, for points A_1, A_2 and A_3, find points respectively $\overline{A_1}\in O_1A_1,\overline{A_2}\in O_2A_2$ and $\overline{A_3}\in O_3A_3$ such that:

$$A_1+\overline{A_1}=O_1;\ A_2+\overline{A_2}=O_2;\ A_3+\overline{A_3}=O_3.$$

Well $\exists\ \overline{(A_1,A_2,A_3)}=\left(\overline{A_1},\overline{A_2},\overline{A_3}\right)\in\mathcal{T}_{\approx}^{\mathcal{D}.Aff}$ such that it:

$$(A_1,A_2,A_3)+\overline{(A_1,A_2,A_3)}=(O_1,O_2,O_3)$$

I summarize what was said earlier in this

Theorem 3.2: *The Groupoid* $\left(\mathcal{T}_{\approx}^{\mathcal{D}.Aff},+\right)$ *is **commutative** (abelian) Group.*

4. The Addition of Similarity *n*-Vertexes in the Desargues Affine Plane

Equally as addition of three-vertexes in Desargues affine plane, by the same logic, additions and n-vertexes in this plane.

Remark 4.1: The set of similarity n-vertexes in Desargues affine plane marked with symbol $\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$.

The addition algorithm of n-vertexes, by analogy with addition algorithm of the three-vertexes are presenting below:

Let's have two whatever similarity n-vertexes in Desargues affine plane:

$$(A_1,A_2,A_3,...,A_n),(B_1,B_2,B_3,...,B_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}.$$

The definitions of the similarity of ***n***-vertexes have the following parallelisms:

$$A_1A_2 \parallel B_1B_2, A_2A_3 \parallel B_2B_3,...,A_{n-1}A_n \parallel B_{n-1}B_n, A_nA_1 \parallel B_nB_1$$

Constructed the lines $A_1B_1,A_2B_2,A_3B_3,...,A_nB_n$, since are in Desargues affine plane, and from the parallels the above, are the conditions of the Desargues theorem, it results that the above lines or crossing from a fixed point **V** or they have a bunch of parallel lines.

In both cases equally found the **zero *n***-vertex. Take one first point $O_1\in A_1B_1$, and then find all the other vertexes of n-vertexes how:

$$O_2=A_2B_2\cap\ell_{A_1A_2}^{O_1},O_3=A_3B_3\cap\ell_{A_1A_3}^{O_1},...,O_n=A_nB_n\cap\ell_{A_1A_n}^{O_1}$$

Definition 4.1: *In the above conditions, application*

$$+:\mathcal{N}_{\approx}^{\mathcal{D}.Aff}\times\mathcal{N}_{\approx}^{\mathcal{D}.Aff}\to\mathcal{N}_{\approx}^{\mathcal{D}.Aff},$$

defined by

$$\left[(A_1,A_2,...,A_n),(B_1,B_2,...,B_n)\right]\mapsto(C_1,C_2,...,C_n)$$

$\forall(A_1,A_2,A_3,...,A_n),(B_1,B_2,B_3,...,B_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *call the addition in* $\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ according to this Definitioni, can write the addition algorithm of the n-vertexes:

$$\forall(A_1,A_2,A_3,...,A_n),(B_1,B_2,B_3,...,B_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$$

$$\left.\begin{array}{l}
1.\ P_1\notin A_1B_1,A_2B_2,\dots,A_nB_n,\\
2.\begin{bmatrix}(i).\ \ell_{A_1B_1}^{P_1}\cap\ell_{O_1P_1}^{A_1}=P_2,\\ (ii).\ \ell_{P_1B_1}^{P_2}\cap A_1B_1=C_1\end{bmatrix}.\\
3.\begin{bmatrix}(i).\ \ell_{A_2B_2}^{P_1}\cap\ell_{O_2P_1}^{A_2}=P_3,\\ (ii).\ell_{P_1B_2}^{P_3}\cap A_2B_2=C_2\end{bmatrix}.\\
4.\begin{bmatrix}(i).\ \ell_{A_3B_3}^{P_1}\cap\ell_{O_3P_1}^{A_3}=P_4,\\ (ii).\ell_{P_1B_3}^{P_4}\cap A_3B_3=C_3.\end{bmatrix}\\
\quad\vdots\\
n+1.\begin{bmatrix}(i).\ \ell_{A_nB_n}^{P_1}\cap\ell_{O_nP_1}^{A_n}=P_{n+1},\\ (ii).\ell_{P_1B_n}^{P_{n+1}}\cap A_nB_n=C_n.\end{bmatrix}
\end{array}\right]\Leftrightarrow \tag{7}$$

$$\Leftrightarrow(A_1,A_2,...,A_n)+(B_1,B_2,...,B_n)=(C_1,C_2,...,C_n).$$

And for n-vertexes, have true analog the statements had to three-vertexes (everything proved equally).

Well have the verities of following statements

Theorem 4.1: *For every two n-vertexes* $(A_1,A_2,...,A_n)$, $(B_1,B_2,...,B_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *, algorithm (7) determines the **single** three-vertexes* $(C_1,C_2,...,C_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *, which does not depend on the choice of hers auxiliary point* P_1.

From Theorem 4.1, appears immediately true this

Proposition 4.1: *Additions of n-vertexes in* $\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *there are element zero the three-vertexes* $(O_1,O_2,...,O_n)$:

$$\begin{aligned}&\forall(A_1,A_2,...,A_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff},\exists(O_1,O_2,...,O_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}\\ &(A_1,A_2,...,A_n)+(O_1,O_2,...,O_n)\\ &=(O_1,O_2,...,O_n)+(A_1,A_2,...,A_n)\\ &=(A_1,A_2,...,A_n)\end{aligned} \tag{8}$$

Also well as worth and below Propositions.

Proposition 4.2: *Additions of n-vertexes is **commutative** in* $\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *:*

$$\forall(A_1,A_2,...,A_n),(B_1,B_2,...,B_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$$

$$\begin{aligned}&(A_1,A_2,...,A_n)+(B_1,B_2,...,B_n)\\ &=(B_1,B_2,...,B_n)+(A_1,A_2,...,A_n)\end{aligned} \tag{9}$$

Proposition 4.3: *Addition of n-vertexes is **associative** in* $\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *:*

$$\forall(A_1,A_2,...,A_n),(B_1,B_2,...,B_n),(C_1,C_2,...,C_n)\in\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$$

$$\begin{aligned}&(A_1,A_2,...,A_n)+\left[(B_1,B_2,...,B_n)+(C_1,C_2,...,C_n)\right]=\\ &=\left[(A_1,A_2,...,A_n)+(B_1,B_2,...,B_n)\right]+(C_1,C_2,...,C_n).\end{aligned} \tag{10}$$

Proposition 4.4: *For every n-vertex in* $\mathcal{N}_{\approx}^{\mathcal{D}.Aff}$ *exists her* ***right symmetrical*** *according to addition:*

$$\forall(A_1, A_2, ..., A_n) \in \mathcal{N}_{\approx}^{\mathcal{D}.Aff}, \exists\overline{(A_1, A_2, ..., A_n)} \in \mathcal{N}_{\approx}^{\mathcal{D}.Aff}$$

$$(A_1, A_2, ..., A_n) + \overline{(A_1, A_2, ..., A_n)} = (O_1, O_2, ..., O_n) \quad (11)$$

(Here have that $\overline{(A_1, A_2, ..., A_n)} = (\overline{A_1}, \overline{A_2}, ..., \overline{A_n})$)

By Theorem 4.1, Propositions 4.1, 4.2, 4.3 and 4.4 we have this true theorem:

Theorem 4.2: *The Groupoid* $(\mathcal{N}_{\approx}^{\mathcal{D}.Aff}, +)$ *is* ***commutative*** *(Abelian) Group.*

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